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PARTICLE DENSITY RETRIEVAL IN RANDOM MEDIA USING A PERCOLATION MODEL AND A PARTICLE SWARM OPTIMIZER

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Particle Density Retrieval in Random Media Using a Percolation Model and a Particle Swarm Optimizer

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Abstract

This letter is a first attempt to apply a percolation theory model to the estimation of the density of particles in complex layered two-dimensional media from electromagnetic measurements. A procedure based on an analytical closed-form description of the wave propagation process is presented. The problem is recast as an iterative optimization one and solved by means of a particle swarm optimizer. Numerical experiments show the validity of the proposed solution.

Index Terms

Percolation theory, Stratified random media, Particle Swarm Optimizers (PSOs), Remote sensing.

I. Introduction

Retrieving the particle density of random media (e.g., hydrometeor masses, granular soils, etc.) is of great interest in several problems arising in remote sensing and radar engineering [1]. This letter proposes a method relying neither on wave theory nor on radiative transfer theory, but describing electromagnetic propagation in terms of a suitable stochastic process [2]. Such a description allows to obtain very simple closed-form analytical solutions that facilitate the inversion procedure. The proposed probabilistic model is a simplified version of the real propagation problem. Nevertheless, it provides the framework for a new inversion method which can be in principle extended to more realistic and complex scenarios. A key assumption is that scatterers are large compared to the wavelength, which is motivated by the recent interest in Terahertz technology [3].

II. Problem Statement

Let us consider a stratified two-dimensional distribution of particles preceded and followed by free-space (Fig. 1) and let us model such a distribution as a two-dimensional percolation lattice [4] described by the following obstacles density profile

$$q(j) = \begin{cases} q_1, & 0 < j \leq l_1 \Rightarrow j \in L_1, \\ q_2, & l_1 < j \leq l_2 \Rightarrow j \in L_2, \\ \vdots & \\ q_K, & l_{K-1} < j \leq l_K \Rightarrow j \in L_K, \end{cases} \tag{1}$$

where each site belonging to level j is independently occupied with probability $q(j)$ (Fig. 1). Each layer L_n is made up by $l_n - l_{n-1}$ levels and its obstacles density is equal to $q_n = 1 - p_n$. Let us assume to know the number of layers K and the depth of each layer. The problem is to estimate the obstacles density values $Q = \{q_n; \, n = 1, ..., K\}$.

We assume to illuminate the half-plane filled by the obstacles by a monochromatic plane wave with free-space power density W_{fs}, that scatterers are large with respect to the wavelength, and that losses and diffractions can be neglected. In this case, the wave can be modeled as a collection of rays undergoing specular reflections on the occupied sites. Hence, the transmitted

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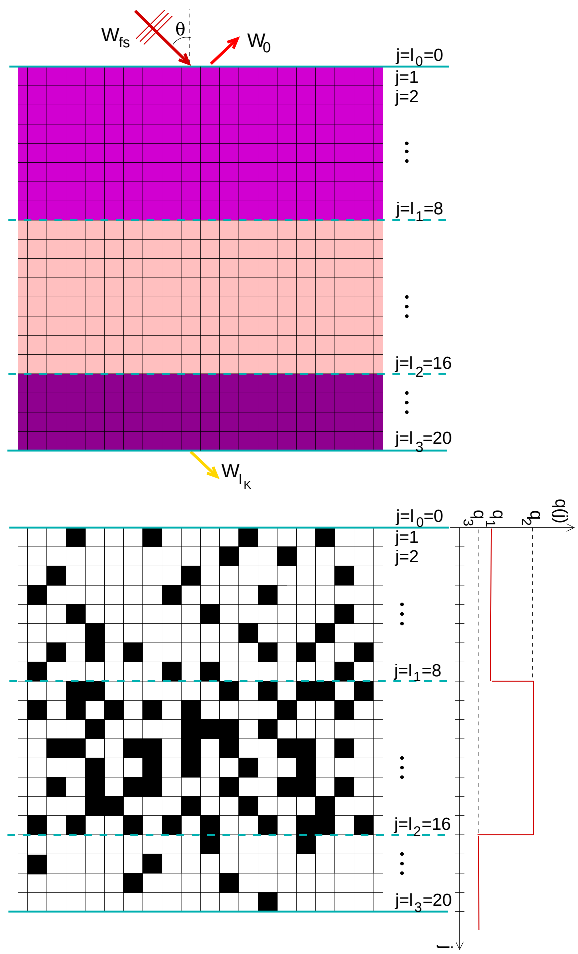

Fig. 1. Sketch of the geometry (top) and the solution (bottom) of the problem.

power density splits along the two vertical directions, see Fig. 1. One portion, W_{l_K}, is associated to the rays crossing the medium and reaching the empty half-plane on the bottom. The other, W_0, is associated to the rays reflected back to the empty half-plane on the top. The values of W_{l_K} and W_0 clearly depend on the unknown characteristics of the medium and carry information on the scatterers density. Let us suppose to measure the backscattered power density W_0.

We now introduce the following notation. We write $\Pr\{A \mapsto B \prec C\}$ to indicate the probability that a ray in level A reaches level B before going into level C. Accordingly, the probability that the ray is reflected back in the above empty half-plane before reaching level l_K is given by [5][6]

$$\Pr\{0 \mapsto 0 \prec l_K\} = 1 - \frac{p_1}{\frac{1}{P_1} + p_1 \sum_{n=2}^{K} \left[\frac{1-P_n}{p_n P_n} + \frac{q_n}{p_n p_{n-1}}\right]}, \tag{2}$$

where $P_n \doteq \Pr\{(l_{n-1}+1) \mapsto l_n \prec (l_{n-1}+1)\}$, $n = 1, ..., K$, is the probability that a ray travels from level $(l_{n-1}+1)$ (i.e., the first level of layer L_n) to level l_n (i.e., the last level of layer L_n) before going back to level $(l_{n-1}+1)$. In the Multilayer Martingale (MMT) approach [5], P_n is estimated as follows:

$$P_n = P_n^{MT} = \begin{cases} 1, & i = 1, \\ \frac{p_n}{q_{e_n} N_n}\left[1 - p_{e_n}^{N_n}\right], & i > 1, \end{cases} \tag{3}$$

where $N_n = (l_n - l_{n-1} - 1)$ and $p_{e_n} = 1 - q_{e_n} = p_n^{\tan\theta+1}$, θ being the incidence angle. In the Markov (MK) approach [6], P_n is estimated as

$$P_n = P_n^{MK} = \frac{p_n}{1 + (N_n - 1)q_n}. \tag{4}$$

The Hybrid (HB) approach proposed in [6] exploits both (3) and (4) as follows: if $q_n < 0.2$, then $P_n = P_n^{MK}$ whatever θ; if $0.2 \leq q_n \leq 0.3$, then $P_n = P_n^{MT}$ if $\theta = 30^o$ and $P_n = P_n^{MK}$ elsewhere; if $q_n > 0.3$, then $P_n = P_n^{MK}$ if $\theta = 15^o$ and $P_n = P_n^{MT}$ otherwise.

Whatever the mathematical approach used, the probability $\Pr\{0 \mapsto 0 \prec l_K\}$ represents the expected number of rays being reflected back in the above empty half-plane. Thus, we have that the backscattered power density is given by

$$W_0(Q) = W_{fs} \Pr\{0 \mapsto 0 \prec l_K\} \left[\frac{W}{m}\right]. \tag{5}$$

Before addressing the inverse problem, we point out the main drawbacks we have to deal with. Besides the typical negative features of the inversion procedures (i.e., non-linearity and ill-posedness), the mathematical models of ray propagation described above satisfactorily perform only in a specific range of parameters. Specifically, as described in [6], more accurate estimations are obtained when the incidence angle tends to 45^o. Furthermore, while the MMT approach and the MK approach perform better for dense and sparse media, respectively, the HB approach allows reliable predictions whatever the obstacles density is [6]. However, the HB approach requires to know the occupation probability of each layer and such a-priori knowledge is not available in this case.

III. Inversion Strategy

The use of the HB approach and of a measurement when the incidence angle θ is equal to 45^o seems to be the best solution to limit the inaccuracies of the models. However, in order to apply the HB approach, the knowledge of the obstacles density distribution is needed. Accordingly, the inversion problem is recast as an iterative optimization problem:

$$Q_E = \arg\left\{\min_i \left[\Theta(Q_i)\right]\right\}, \tag{6}$$

where Q_E is the estimated obstacles density profile, $\{Q_i;\, i = 1, ..., I\}$ is a sequence of trial solutions, i being the iteration number, and

$$\Theta(Q_i) = \Theta_{HB}(Q_i) \doteq \frac{\left|[W_0(Q_i)]_{HB}^{45^o} - [W_0]_m^{45^o}\right|}{[W_0]_m^{45^o}}, \tag{7}$$

where $[W_0(Q_i)]_{HB}^{45^o}$ and $[W_0]_m^{45^o}$ are the estimated and measured power density values, respectively.

The ill-posedness of the problem is due to the loss of information in the solution of the forward problem, where an input quantity (i.e., $Q = \{q_n;\, n = 1, ..., K\}$) is mapped into an output value (i.e., the power density W_0) with a smaller information content. The most natural way to introduce information would be to consider additional incidence conditions besides $\theta = 45^o$. However, as noticed before, the accuracy of the mathematical models decreases as θ deviates from 45^o. Hence, we exploit a new model when the probing wave impinges normally on the half-plane lattice (i.e., $\theta = 0^o$). We refer to this in the following as Normal Incidence (NI) approach. Let us consider a single ray entering the grid and let us estimate the probability $\Pr\{0 \mapsto 0 \prec l_K\}$. Such a quantity is equal to the probability that any of the l_K cells on the ray path is occupied. As a matter of fact, if a reflection occurs, such a reflection is surely on a horizontal face and the ray escapes from the grid traveling along the free path just covered, but with negative direction. Thus, provided that $\theta = 0^o$, the backscattered power density is given by

$$[W_0(Q_i)]_{NI}^{0^o} = W_{fs}\left[1 - \prod_{n=1}^{k} p_n^{S_n}\right], \tag{8}$$

where $S_n = l_n - l_{n-1}$. Accordingly, the functional to be minimized takes the form

$$\Theta(Q_i) = \Theta_{HB,NI}(Q_i) \doteq \Theta_{HB}(Q_i) + \Theta_{NI}(Q_i), \tag{9}$$

where

$$\Theta_{NI}(Q_i) \doteq \frac{\left|[W_0(Q_i)]_{NI}^{0^o} - [W_0]_m^{0^o}\right|}{[W_0]_m^{0^o}}. \tag{10}$$

Moreover, some additional information on the actual solution is introduced by exploiting the *phase transition property* exhibited by percolation lattices [4]. According to such a property, propagation is inhibited when p_n, $n = 1, ..., K$, is lower than the so-called *percolation threshold* p_c, $p_c \approx 0.59275$ in the two-dimensional case. Thus, when $p_n < p_c$, the backscattered value W_0 tends to W_{fs} and it is not possible to extract any reliable information on the medium at hand from the field measurement. Accordingly, when looking for the medium distribution, we can set

$$p_c < p_n \leq 1, \; n = 1, ..., K. \tag{11}$$

In order to look for the global minimum of (9) that satisfies (11), an optimization algorithm able to effectively explore the solution space is needed. In such a choice, the non-linearity of the problem plays a relevant role. Although some *a-priori* information has been introduced, the cost function still presents several local minima, which correspond to false solutions of the physical problem. Moreover, (9) has some discontinuities. To overcome these drawbacks, a typical solution is to use global optimization techniques, such as Genetic Algorithms (GAs) [8] and Particle Swarm Optimizers (PSOs) [9]. In fact, deterministic approaches such as gradient methods [10] are reliable only when the cost function is everywhere differentiable and the search space is limited at the attraction basin of the global minimum.

A PSO is applied here. The choice has been motivated by the advantages exhibited by PSOs when compared to GAs. Such advantages are mainly concerned with the ability to control the convergence and the stagnation of the optimization process, an easier implementation and calibration, and the exploitation of the cooperation among the trial solutions. Moreover, PSOs present a better heuristic adaptability with respect to GAs, where stagnation phenomena can be avoided only thanks to lucky mutations. In the following, the main steps of the implemented PSO are summarized.

Initialization Step ($i = 0$). The positions of the P particles of the swarm $Q_{0,p} = \{(q_n)_{0,p};\; n = 1, ..., K\}$, $(q_n)_{0,p} \in [0, 1-p_c)$, and their velocities $V_{0,p} = \{(v_n)_{0,p};\; n = 1, ..., K\}$, $p = 1, ..., P$, are randomly generated.

Evaluation Step. The optimality of each trial solution at the i-th iteration is evaluated and the *personal best* position

$$B_{i,p} = \{(b_n)_{i,p};\; n = 1, ..., K\} = \arg\left\{\min_{h=0,...,i}\left[\Theta(Q_{i,p})\right]\right\} \tag{12}$$

as well as the *global best* position

$$G_i = \{(g_n)_i;\; n = 1, ..., K\} = \arg\left\{\min_{p=1,...,P}\left[\Theta(B_{i,p})\right]\right\} \tag{13}$$

are updated. The iteration index is increased ($i = i + 1$) and the termination criteria are checked. If the cost of the global best is smaller than a given threshold η or the maximum number of iteration I is reached, then the optimization process stops and the global best is assumed as the problem solution Q_E.

Updating Step. The velocity of each particle is updated:

$$\begin{aligned}(v_n)_{i,p} = \omega(v_n)_{i-1,p} + c_1\rho_1\left[(b_n)_{i-1,p} - (q_n)_{i-1,p}\right] \\ +c_2\rho_2\left[(g_n)_{i-1,p} - (q_n)_{i-1,p}\right],\end{aligned} \tag{14}$$

where ω, c_1 and c_2 are constant parameters called *inertial weight*, *cognition* and *social acceleration*, respectively. Moreover, ρ_1 and ρ_2 are random coefficients drawn from a uniform distribution in [0,1]. The position of each particle is updated as follows

$$(q_n)_{i,p} = (q_n)_{i-1,p} + (v_n)_{i,p}. \tag{15}$$

Particles escaping the actual solution space are handled according to the reflecting wall technique [11]: whenever the particle hits the boundary of the solution space along direction n, then the sign of the velocity in such direction is changed and the particle is reflected back in the solution space. The optimization algorithm restarts from the "**Evaluation Step**".

IV. Numerical Validation

The proposed inversion strategy is validated by referring to three-layer profiles having $l_1 = 8$, $l_2 = 16$ and $l_3 = 20$ (Fig. 1). Such a configuration could be of interest to model a rain column, which is usually considered as made by three regions [7]. More in detail, experiments consider three different test cases, i.e., a profile consisting of very sparse and very dense layers in alternated succession, $Q_R = \{0.05, 0.35, 0.05\}$, a sparse profile, $Q_R = \{0.05, 0.15, 0.05\}$, and a dense profile, $Q_R = \{0.35, 0.25, 0.35\}$.

The scattering data $[W_0]_m^{45^o}$ and $[W_0]_m^{0^o}$ have been numerically obtained by Monte Carlo computer-based ray tracing experiments. Specifically, 100 random lattices with the same obstacles density have been generated and for every grid 500 rays have been launched from different entry positions.

The PSO parameters values are given in Table I and have been chosen following the guidelines provided in [12]. In particular, considering the dimension of the solution space and in order to avoid not strictly necessary fitness evaluations, P has been set equal to 5. The parameters I and η have been empirically chosen. The inertial weight ω has been set equal to 0.4 to damp oscillations of the optimizer around the optimal solution and speed up the convergence rate, while c_1 and c_2 have been set equal to 2 [12]. Taking into account the dynamic range of the particle, V_{max} has been set equal to 0.4. For each experiment, the optimizer has been executed $T = 10$ times.

TABLE I

PSO SETUP PARAMETERS.

P	I	η	ω	c_1	c_2	V_{max}
5	2000	10^{-5}	0.4	2.0	2.0	0.4

In order to quantify the effectiveness of the inversion procedure, the *discrepancy* $\Delta = \left[\frac{1}{K}\sum_{n=1}^{K}\left|(q_n)_R - (q_n)_E\right|\right] \times 100$ is analyzed, $(q_n)_R$ and $(q_n)_E$ being the reference and the estimated occupation probability values, respectively. More in detail, since the PSO is executed T times for each experiment, the average value $\Delta_{av} = \frac{1}{T}\sum_{t=1}^{T}\Delta_t$, the standard deviation $\sigma_\Delta = \frac{1}{T}\sum_{t=1}^{T}\left|\Delta_t - \Delta_{av}\right|$, the maximum $\Delta_{max} = \max_t\{\Delta_t\}$, and minimum $\Delta_{min} = \min_t\{\Delta_t\}$ are evaluated, Δ_t being the discrepancy obtained at the t-th trial.

The proposed approach allows good estimations of the unknown probability distributions (Fig. 2). This is confirmed by the discrepancy values, being $\Delta_{av} = 2.72\%$, $\Delta_{av} = 2.54\%$, and $\Delta_{av} = 6.11\%$, for the variable, the sparse and the dense profiles, respectively. It is worth noting that obtained results considerably worsen when $Q_R = \{0.35, 0.25, 0.35\}$. In fact, at higher densities rays are almost immediately backscattered without exploring much of the medium and therefore the measured power density does not carry much information useful for the inversion procedure.

For comparison purposes, we consider results obtained by minimizing other kinds of cost functions, i.e., (7) and

$$\begin{aligned}\Theta_{MMT,MK,NI}(Q_i) \doteq \Theta_{MMT}(Q_i) + \\ \Theta_{MK}(Q_i) + \Theta_{NI}(Q_i),\end{aligned} \tag{16}$$

where $\Theta_{NI}(Q_i)$ is given in (10) and

$$\Theta_{MMT}(Q_i) \doteq \frac{\left|[W_0(Q_i)]_{MMT}^{45^o} - [W_0]_m^{45^o}\right|}{[W^0]_m^{45^o}}, \tag{17}$$

$$\Theta_{MK}(Q_i) \doteq \frac{\left|[W_0(Q_i)]_{MK}^{45^o} - [W_0]_m^{45^o}\right|}{[W_0]_m^{45^o}}. \tag{18}$$

The underlying idea of (16) is combining the two different terms $\Theta_{MMT}(Q_i)$ and $\Theta_{MK}(Q_i)$ in order to compensate the complementary negative features of the MMT approach and of the MK approach, which satisfactorily perform only provided that the obstacles density is high and low, respectively.

Comparing the proposed strategy with the one relying on $\Theta_{HB}(Q_i)$ (Fig. 3), it is evident that minimizing $\Theta_{HB,NI}(Q_i)$ leads to better estimations (for instance, $\frac{[\Delta_{av}]_{HB}}{[\Delta_{av}]_{HB,NI}} \cong 6.2$ when $Q_R = \{0.05, 0.35, 0.05\}$). Such a behavior validates the effectiveness of introducing the additional term $\Theta_{NI}(Q_i)$ in the cost function to be minimized.

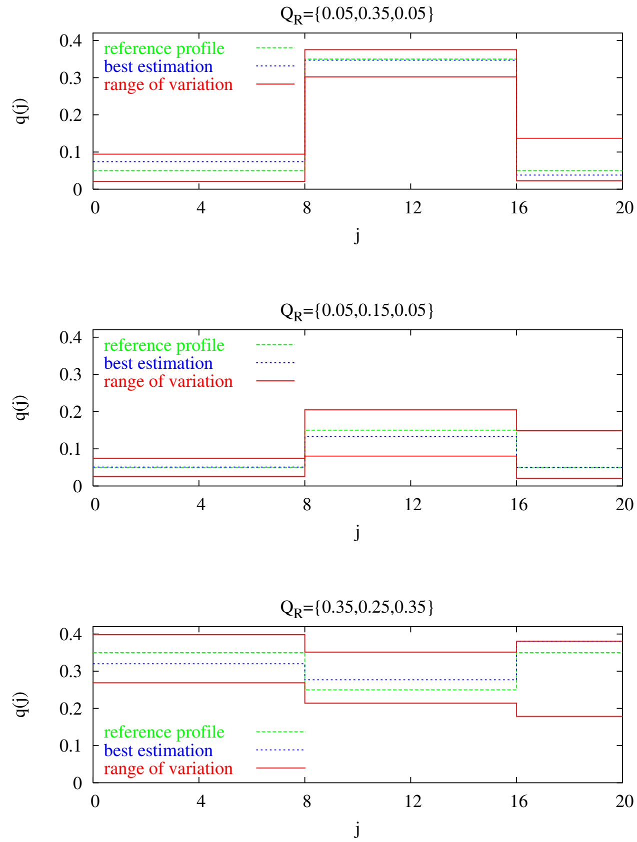

Fig. 2. Retrieved obstacles density profiles.

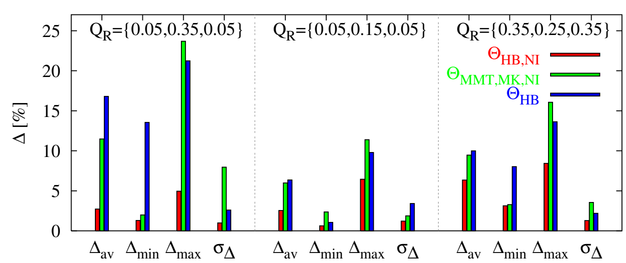

Fig. 3. Error statistics.

The proposed strategy outperforms the one relying on (16) as well (Fig. 3). This is particularly evident when $Q_R = \{0.05, 0.35, 0.05\}$, being $\frac{[\Delta_{av}]_{MMT,MK,NI}}{[\Delta_{av}]_{HB,.NI}} \cong 4.2$. Such a behavior can be explained taking into account that neither the MK approach nor the MMT approach satisfactorily perform in describing ray propagation when highly variable profiles are at hand. On the contrary, when either dense or sparse profiles are considered, performances of the strategy relying on $\Theta_{MMT,MK,NI}(Q_i)$ get better since one between the MK approach and the MMT approach properly works.

V. Conclusions

In this letter, a new approach to the retrieval of the density of particles in complex layered media from electromagnetic measurements has been proposed. Thanks to the analytical nature of the model estimating the measured power density, the convergence rate of the PSO (i.e., satisfactory solutions are obtained after few hundreds of iterations), and the very small ensemble of unknowns, the proposed inversion procedure turns out to be extremely fast. Numerical experiments have shown that reliable estimations can be achieved.

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